Development of Absorbing Conditions for the Analysis of Finite Dimension Elastic Wave-Guides

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***Abstract*—A perfectly matched layer approach was developed for piezoelectric problems simulated by finite element and compared with a distributed viscosity absorption method. The development is self-consistent and allows for absorbing energy at the edges of a mesh without spurious reflection due to non appropriated boundary conditions. 1D and 2D validation are reported and the method is applied to analyze a finite length Lamb wave transducer and a 2D FBAR structure.**

I. Introduction

The development of acoustic wave guides for signal processing has benefited from a huge effort in setting up advanced simulation tools combining analytical and numerical analyses. Surface Acoustic wave (SAW) devices are mainly based on periodic structures allowing for a very accurate definition of the resonance frequency, but other principles such as Film Bulk Acoustic Resonator (FBAR) are presently used to fabricate Radio-Frequency (RF) filters and ultra compact sources. In the latter case, the ultimate optimization will take advantage of the capability to simulate the actual form of the resonator. In the case of SAW devices, the main effort is presently dedicated to account for lateral resonance that may pollute the main SAW contribution exploited for the filter operation. Such phenomena can be correctly modeled using advanced numerical tools, and more particularly, a formulation has to be developed to allow for only analyzing the active part of the resonator plus the area close to the edges, and suppressing modes generated by the lack of consistent absorbing conditions.

In that matter, we have developed two different approaches based on our periodic Finite Element Analysis (FEA) code including radiation boundary conditions. The first one consists in a distributed visco-elastic absorption on the edge of the device. This approach is physically consistent and is expected to minimize the amount of reflected energy thanks to a smoothly varying absorption distribution. The other approach is based on the Perfectly Matched Layer (PML) concept first developed by Bérenger in 1994 [1] to address electromagnetism problems. A lot of work has been devoted to transpose this approach to elasticity problems [2, 3, 4]. It can be understood along different way, but it basically consists in a mathematics-based astuteness for describing a localized domain (e.g. an acoustic transducer) radiating energy in open surrounding media (no reflection should occur at the boundary of the considered radiating domain). The PML thus must absorb any energy entering it from the above mentioned interface and should not generate any contribution within the said transducer area.

In this presentation, we propose to compare both approach for piezoelectricity-based problems and we show how one has to change the material constants to ensure the operation of both methods. A first numerical application is achieved in 2D to address the simulation of edge effects in 2D FBARs for which the influence of layer arrangement may significantly influence the operation, and also for a finite length Lamb mode transducer on quartz. As a conclusion, we propose different tracks to further develop the PML approach, particularly for 3D piezoelectric problems.

II. Fundamentals

Distributed visco-elasticity

The first intuitive approach to develop absorbing conditions at the edge of a FEA mesh consists in smoothly varying visco-elastic constants when getting close to the said edges, to avoid sharp acoustic impedance changes yielding reflections toward the meshed transducer. This is quite easy to achieve in FEA by introducing in the Lagrangian a frequency dependent term accounting for Newtonian visco-elastic effects

$$\sum_{e=1}^{E}\iiint_{\Omega_e}\left(-\rho_e\omega^2\delta u_i^{*e}u_i^e+\frac{\partial\delta u_i^{*e}}{\partial x_j}\left(C_{ijkl}^e+j\omega d(x_1)\eta_{ijkl}^e\right)\frac{\partial u_l^e}{\partial x_k}\right)dV=\sum_{e=1}^{E}\iint_{\Gamma_e}\delta u_i^{*e}T_{ij}^e n_j dS \quad (1)$$

This formulation actually corresponds to a problem for which the absorbing area exhibits visco-elastic losses increasing along the x_1 axis following the law $d(x_1)$. This later function is defined along the same rules as those considered for PMLs (see next section). This approach was implemented for instance by Reinhardt [9] to address the problem of FBARs. Its main advantage is the preservation of symmetrical matrices in the final digitalized FEA system, which is not the

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case for PML as it will be shown later. However, the acoustic impedance still varies along the absorbing elements and energy still can be reflected in given cases. 1D and 2D FEA have been easily implemented to test the efficiency of the approach, as there is no fundamental changes in the FEA formulation but adding frequency dependent visco-elastic losses.

Perfectly Matched Layers

The basic idea consists in rigorously simulating an exponential decrease of the acoustic field along at least one space direction. To clear the approach, let us consider the following incident plane wave

$$u = Ae^{-j(k_x x - \omega t)} = Ae^{-j\omega(s_x x - t)} \quad (2)$$

We then consider that in the absorbing area, one can apply a geometrical transform in the complex plane to introduce the exponential decay. Since it must not modify the propagation phase, this transform can be written

$$\tilde{x} = x - j.f(x) \quad (3)$$

where f(x) growths from the origin of the absorbing area to its end along a defined rate. However, since this transform must be efficient for any frequency (we represent the problem in the spectral domain), it is wise to define this function as follows

$$f(x) = \frac{1}{\omega}\int_0^x d(n)dn \quad (4)$$

which allows for an easy definition of the transform Jacobian linking the considered coordinate systems. This reads

$$\frac{1}{\partial\tilde{x}} = \frac{j\omega}{j\omega + d(x)}\frac{1}{\partial x} \rightarrow \frac{1}{\partial x} = \left(1 + \frac{d(x)}{j\omega}\right)\frac{1}{\partial\tilde{x}} = \alpha_x \frac{1}{\partial\tilde{x}} \quad (5)$$

Replacing x by $\tilde{x}$ in (2) provides the wanted exponential decay if $f(x)$ unconditionally growths, imposing $d(x)$ even and positive to fulfil the absorbing condition for any x (we assume the problem centred around x=0). Since the absorbing function $d(x)$ is not frequency dependent, its efficiency should be constant along ω. Conformably to Zheng and Huang [3], we develop a formulation based on the usual piezoelectricity equations, yielding significant modifications of the elastic, piezoelectric and dielectric constants to account for the absorption.

We now rewrite the elasticity equations in the absorbing region turning x to $\tilde{x}$, using then (5) to express the result in the initial coordinates. As in [3], the absorbing effect is assumed along the three space directions for the sake of generality. The equilibrium equation then reads

$$-\rho\omega^2 u_i = \frac{\partial T_{ij}}{\partial\tilde{x}_j} = \frac{1}{\alpha_j}\frac{\partial T_{ij}}{\partial x_j} \quad (6)$$

where α_j is characterised by its specific function $d_j(x_j)$. T_{ij} and u_i respectively represent the dynamic stresses and displacements, and ρ is the mass density. We introduce a non symmetrical stress tensor, expressed in the transformed axis

$$\tilde{T}_{ij} = \frac{\alpha_1\alpha_2\alpha_3}{\alpha_j}C_{ijkl}\frac{\partial u_l}{\partial\tilde{x}_k} = \frac{\alpha_1\alpha_2\alpha_3}{\alpha_j\alpha_k}C_{ijkl}\frac{\partial u_l}{\partial x_k} = \tilde{C}_{ijkl}\frac{\partial u_l}{\partial x_k} \quad (7)$$

where $\tilde{C}_{ijkl}$ is the transformed elastic constant tensor relative to the absorption area. We multiply (6) by $\alpha_1\alpha_2\alpha_3$, thus yielding Newton relation for PMLs in the real coordinates

$$-\tilde{\rho}\omega^2 u_i = \frac{\partial\tilde{T}_{ij}}{\partial x_j} \quad (8)$$

where $\tilde{\rho} = \rho\alpha_1\alpha_2\alpha_3$ is the mass density relative to the transformed domain. Since the obtained form of the equilibrium equation complies with the classical expression for usual solids, it is liable to exploit the standard FEA formulation for PML as well, accounting for the frequency dependence of the transformed physical tensors. These developments of course can be extended to piezoelectricity by rewriting Poisson's equation and taking into account the piezoelectric coupling in the stress definition as follows

$$\tilde{T}_{ij} = \frac{\alpha_1\alpha_2\alpha_3}{\alpha_j}\left(C_{ijkl}\frac{\partial u_l}{\partial\tilde{x}_k} + e_{kij}\frac{\partial\phi}{\partial\tilde{x}_k}\right) = \tilde{C}_{ijkl}\frac{\partial u_l}{\partial x_k} + \tilde{e}_{kij}\frac{\partial\phi}{\partial x_k} \quad (9)$$

Poisson's equation expressed in the transformed system of axes reads

$$\frac{\partial D_i}{\partial\tilde{x}_i} = 0 \Rightarrow \frac{1}{\alpha_i}\frac{\partial D_i}{\partial x_i} = 0 \quad (10)$$

with D_i the electrical displacement vector. To provide an homogeneous formulation, we proceed as for the stress definition (7), multiplying the electrical displacement by $\alpha_1\alpha_2\alpha_3$, yielding

$$\tilde{D}_i = \frac{\alpha_1\alpha_2\alpha_3}{\alpha_i}D_i \quad (11)$$

As for the propagation equation (6), the Poisson's condition is written accounting for these changes as

$$\frac{\partial\tilde{D}_i}{\partial x_i} = 0 \quad (12)$$

Conformably to the stress tensor transformation, we introduce modified piezoelectric and dielectric constants defined as follows

$$\tilde{D}_i = \frac{\alpha_1\alpha_2\alpha_3}{\alpha_i}\left(e_{ikl}\frac{\partial u_l}{\partial\tilde{x}_k} - \varepsilon_{ik}\frac{\partial\phi}{\partial\tilde{x}_k}\right) = \tilde{e}_{ikl}\frac{\partial u_l}{\partial x_k} - \tilde{\varepsilon}_{ik}\frac{\partial\phi}{\partial x_k} \quad (13)$$

We now are able to establish a FEA formulation exploiting these developments without fundamental changes of the existing code. As some care must be paid to the choice of the absorption function, the next paragraph is dedicated to that question.

Absorption function

In our developments, the absorption function is chosen to avoid introducing any brutal change in the physical constants. This signifies it must exhibits zero derivatives at each edge of the PML area, but it must continuously vary more rapidly within this area to avoid meshing a too long PML zone. We also want this function to be even (symmetrical) so it can apply at each edge of a domain centred around x=0. As proposed in [4], we consider the following form of $d(x)$ that allows to match all these specifications at once

$$d(x_1) = \begin{cases} d_{\max}\left(1 - \dfrac{(|x_1| - x_p)^2}{(x_a - x_p)^2}\right)^n & \rightarrow x_a < |x_1| < x_p \\ 0 & \rightarrow |x_1| < x_a \end{cases} \tag{14}$$

with x_a and x_p the limits of the PML area and n an integer controlling the absorption rate. The coefficient d_{max} is an adjustable normalisation parameter. This function is plotted in fig.1 to show its compliance to the above definition. For practical application, we chose n=3 as the best trade-off between a strong absorption rate and fast computations, as the PML must be computed for each frequency point.

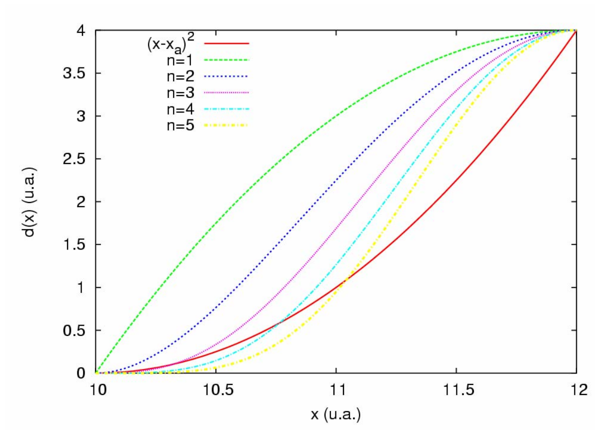

Figure 1. Comparison between different absorption rate of the function $d(x)$

III. NUMERICAL TESTS AND VALIDATION

We have considered the simple case of a 1mm thick 1D PZT transducer surrounded by two low acoustic impedance semi-infinite media (lens) for a first validation of our developments. This configuration allows for an easy identification of the PZT longitudinal mode resonance and the dissipation effects due to radiation in the semi-infinite media. Furthermore, this calculation is easily achieved using the analytical model described in [5]. We have implemented this simulation using the proposed theoretical development restricted to 1D problem. Absorption occurs at the edge of the mesh plotted in fig.2 (the blue area corresponds to the PML or to the distributed visco-elasticity). The red central bar represents the PZT on which a unit potential excitation is applied and the green sections corresponds to intermediate layer coupling the PZT with absorption areas.

Figure 2. 1D mesh for the first validation computation

Figure 3 shows the transducer conductance, comparing the exact calculation results to those obtained using distributed visco-elasticity for various absorption coefficient d_{max}. Unwanted resonance still persists before and after the longitudinal mode resonance of the transducer.

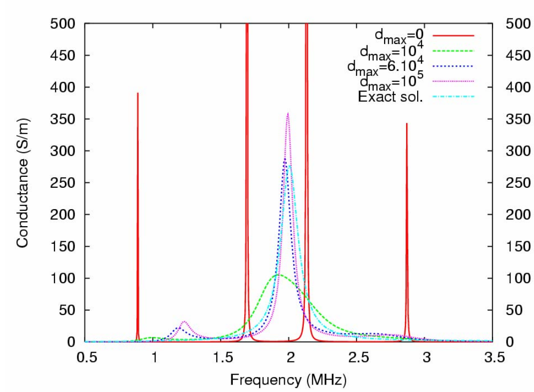

Figure 3. Comparison between the exact solution and distributed visco-elastuc absorption simulation of a 1D PZT transducer radiating in semi-infinite solids

The transducer conductance has been computed with our PML formulation and compared to the exact solution, the best results being achieved for $10^5 < d_{max} < 10^6$. Notable discrepancies appear for the amplitude and more particularly the width of the conductance peak of the exact solution and PML computations. Although the PML approach accurately simulates the acoustic radiation, it generates losses in excess compared with the exact problem.

The same computations have been performed using 2D 3-node triangles. A 2D mesh of the transducer has been implemented (fig.5), exhibiting the same length as the 1D mesh but with various ridge width to check the efficiency and the stability of the formulation. Figure 5 compares the 1D and 2D PML approaches with the exact solution. One still can note that for both PML computation, the width of the conductance is larger than for the exact solution. The later assumes infinite (1D) radiating surfaces. This significant difference with the FEA computations may explain the discrepancies between thos results. Whatever, it is proved that the PML approach is able to accurately simulate lateral radiations in amore efficient way than the distributed visco-elastic absorption does.

Figure 4. 2D mesh of the 1D problem

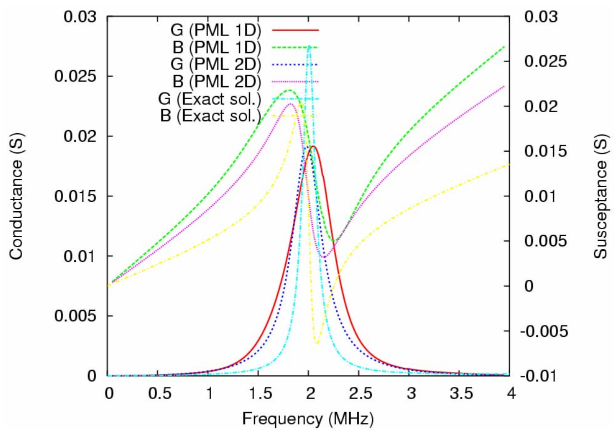

Figure 5. Comparison between the exact solution and 1D and 2D PML-based simulations of the PZT transducer radiating in semi-infinite solids

After validating the PML formulation, we have investigated the applicability of the approach to the simulation of actual 2D finite length device. The first one is a Lamb wave inter-digital transducer exhibiting 16 finger pairs deposited atop AT quartz. The corresponding mesh is plotted in fig.7, the number of nodes is 6376 (11209 elements) with 3 degree-of-freedom (d.o.f.) per node as the polarization of the modes is mainly elliptic on this quartz cut. The mechanical period of the electrode grating is 20µm, and the metal ratio is 0.5. Aluminum electrodes also have been meshed (*h*/2*p*=0.25).

Figure 6. 2D mesh of the 16 finger-pair Lamb wave transducer on Quartz

We have computed the transducer response considering first no absorption in the blue edging area, and then applying the PML. Figure 7 shows the superimposition of both computations results. To higlight the actual role of the PML, reduced acoustic loss was considered within the plate. Consequently, the main source of insertion loss relies in the edge absorption. As expected, the classical sin(*x*)/*x* function is found when using PML. This is the main proof of the efficient PML operation (10 PML).

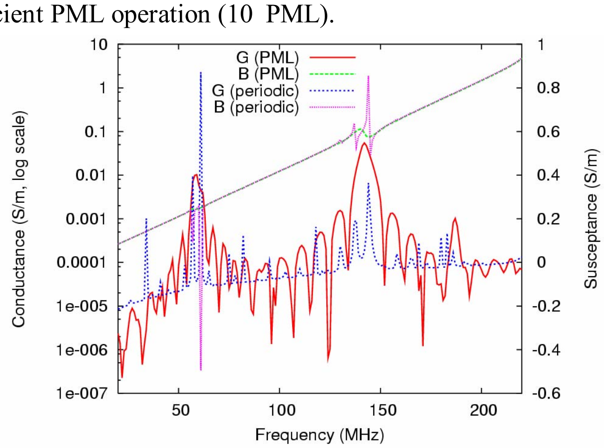

Figure 7. Comparison between the exact solution and 1D and 2D PML-based simulations of the PZT transducer radiating in semi-infinite solids

The last result of this paper concern a membrane-based FBAR operating near 2 GHz. The same procedure than the one applied for the Lamb wave device was considered here, yielding results reported in fig.8. The operation of the PML is here again very clear for frequencies below the fundamental (flexural modes), and it is found not affecting the main FBAR resonance, which was one of the main requirement of this work.

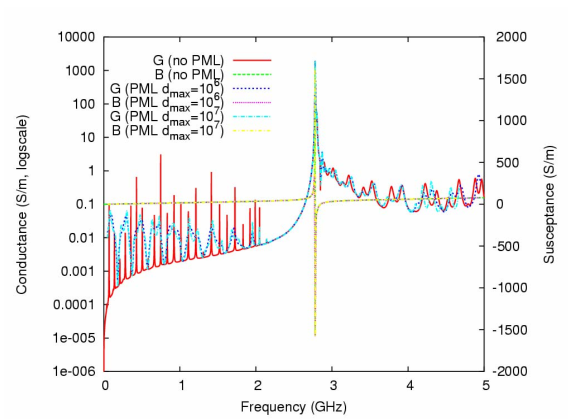

Figure 8. FBAR response without and with PML absorbing conditions

IV. CONCLUSION

PMLs have been successfully implemented in 1D and 2D to address the simulation of finite length transducers assume to laterally radite energy.. It has been compared to a distributed visco-elastic absorbing condition which does not fill all the expected requirements. Consequently, only the PML approach was found to answer the demand, yielding realistic simulations of Lamb wave transducer and a simple FBAR. Further developments will be devoted to 3D problems together with frequency acceleration of the code, to compensate the computation of finite element stiffness and mass matrices for each frequency.

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